QUADS

Quantum Angular Density Substrate

Paper 1

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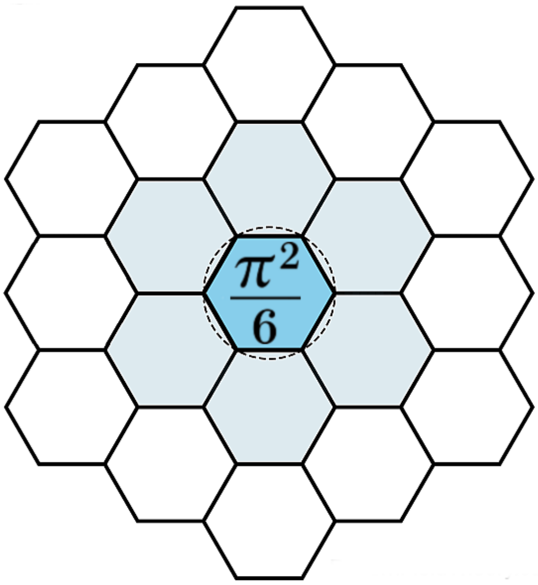


Abstract

We construct the **Quantum Angular Density Substrate (QUADS)** as a transport-first physical ontology. The substrate consists of discrete angular density elements distributed across lateral adjacency relations and inward expansion depth layers. Physical persistence, locality, and conservation arise from participation structure rather than from a presupposed spacetime manifold.

Axioms governing substrate existence, participation density, transport conservation, and dynamical extremization are stated explicitly. No metric, curvature tensor, or geometric manifold is assumed at this stage. The framework establishes the primitive ontological layer upon which nonlinear dynamics and emergent geometry are constructed in subsequent work.

Foundational Structural Precedents

The Quantum Angular Density Substrate (QUADS) framework builds upon prior structural investigations addressing dimensional stratification, scale representation, and morphogenic stabilization.

Three precursor studies establish the mathematical and geometric foundations integrated in this work:

1. **Logarithmic Dimensional Shift** - demonstrating how layered manifold structure can arise from branch discontinuities and discrete lifting.
2. **Logarithms as Geometric Charts** - formalizing scale transitions as coordinate chart transformations across stratified participation layers.
3. **Six-Fold Morphogenics** - establishing how persistent structural form arises through multi-layer stabilization of patterned fields.

These works collectively provide the dimensional, representational, and morphogenic mechanisms required for a transport-based physical substrate to yield stable relational invariants.

QUADS I integrates these structural mechanisms into a unified ontological substrate defined by participation density and angular transport conservation.

Terminology and Acronym Expansion

Quantum Angular Density Substrate (QUADS) denotes the fundamental physical substrate composed of discrete angular density elements distributed across lateral adjacency structure and inward expansion depth.

Quantum Angular Density Dynamics (QUAD) denotes the dynamical theory governing the evolution of QUADS.

Allen Orbital Lattice (AOL) denotes the structural lattice realization of QUADS.

All terminology is defined operationally. Conceptual interpretation is distinguished from formal derivation throughout.

Ontological Identification

Substrate Identity

$$\text{QUADS} := \text{Quantum Angular Density Substrate}$$

$$\text{AOL} := \text{structural realization of QUADS}$$

$$\text{QUAD} := \text{dynamical laws governing QUADS}$$

The Allen Orbital Lattice (AOL) is the structural realization of the QUADS. QUADS constitutes the physical substrate. QUAD specifies the dynamical law governing the evolution of QUADS.

No metric manifold or geometric tensor structure is assumed as primitive.

Axiomatic Foundation of QUADS

Axiom 1 - Substrate Existence

A physical substrate exists consisting of discrete angular transport capacity distributed across finite lateral adjacency relations and inward expansion depth layers.

Axiom 2 - Participation Density

Local physical state is determined by a scalar participation density $D(x)$ representing active inward expansion engagement at substrate location x.

Axiom 3 - Transport Conservation

Angular transport capacity is locally conserved under closure ordering. No angular density is created or destroyed except through defined participation processes.

Axiom 4 - Dynamical Extremization

Physical evolution extremizes a participation action functional $S[\psi]$ defined over participation amplitude $\psi(x)$, where

$$\psi(x) = \sqrt{D(x)}.$$

Axiom 5 - Emergent Relational Structure

Metric and geometric relations arise from spatial variation of participation amplitude. They are not primitive properties of the substrate.

Axiom 6 - Observability

All measurable physical quantities depend exclusively on participation distribution or its evolution.

Participation Density and Amplitude Formalization

The primitive physical state of the Quantum Angular Density Substrate (QUADS) is described by a non-negative participation density field:

$$D : X \to \mathbb{R}_{\geq 0},$$

where X denotes the discrete substrate index set consisting of lateral adjacency relations and inward expansion depth.

Participation Amplitude

Define the participation amplitude:

$$\psi(x) = \sqrt{D(x)}.$$

This definition ensures:

1. $D(x) \geq 0$ by construction,
2. additive structure in transport expressions,

3. compatibility with quadratic action functionals.

Participation amplitude is not a wavefunction. It is a structural representation of engagement intensity.

Local Participation Measure

For any finite region $\Omega \subset X$, define total participation:

$$\mathcal{P}(\Omega) = \sum_{x \in \Omega} D(x).$$

This quantity represents aggregate engagement capacity within Ω.

x-Register Ontology

To formalize minimal participation events, we introduce the x-register.

Definition 1 (x-Register)**.** *An x-register is the minimal discrete state carrier of participation, admitting the tri-state space:*

$$S = \{0, null, 1\}.$$

Where:

- 1 denotes affirmed participation outcome,
- 0 denotes negated participation outcome,
- null denotes structurally open participation.

Null as Participation Openness

Null is not absence. Null is an unoccupied participation placeholder representing:

- potential outcome capacity,
- admissible possibility set,
- pre-resolution probability structure.

Null is therefore the necessary structural condition for state transformation.

Proposition 1. *No x-register transition can occur without passage through null.*

Justification.

Resolved states $\{0, 1\}$ represent stabilized participation. Transformation requires participation openness. Therefore null is the required intermediary state.

Minimal Observance Structure

Observance is defined structurally, independent of cognition.

Definition 2 (Observance Event)**.** *An observance event consists of:*

1. *Acknowledgement - selection of an x-register,*
2. *Coupling - interaction modifying or stabilizing the register state.*

Observance is therefore the minimal mechanism by which participation state becomes updated or reinforced.

Participation Update Rule

Let $x \in S$ denote register state.

Minimal update schema:

$$x(t+1) = \begin{cases} 1 & \text{if coupling resolves null positively,} \\ 0 & \text{if coupling resolves null negatively,} \\ x(t) & \text{if insufficient coupling occurs.} \end{cases}$$

This preserves:

- structural openness prior to resolution,
- stabilization after resolution,
- conservation of participation transitions.

Participation and Conservation Compatibility

Participation density $D(x)$ and x-register states are compatible through mapping:

$$D(x) = \sum_{i \in \mathcal{R}(x)} \delta_{s_i,1},$$

where $\mathcal{R}(x)$ denotes registers associated with substrate location x, and δ is the Kronecker delta.

Participation density therefore represents aggregated affirmed registers.

Transport conservation operates over register transitions collectively, ensuring no net creation of participation without defined coupling.

Remark 1. *The x-register formalism provides the minimal discrete ontology required for participation update while remaining compatible with continuous participation density representations.*

Transport Conservation and Closure Ordering

The Quantum Angular Density Substrate (QUADS) is governed by local transport conservation.

Let $x \in X$ denote substrate locations and $\mathcal{N}(x)$ the adjacency set.

Discrete Transport Flux

Define transport flux between adjacent locations:

$$J_{x \to y} = \kappa \left(\psi(y) - \psi(x) \right),$$

for $y \in \mathcal{N}(x)$ and $\kappa > 0$.

Local Conservation Law

Participation transport satisfies:

$$\sum_{y \in \mathcal{N}(x)} J_{x \to y} = 0.$$

This ensures that participation capacity is redistributed but not created or destroyed through adjacency transport alone.

Proposition 2. *Transport conservation preserves total participation over finite substrate regions in the absence of external coupling.*

Justification.

Summing over a finite region Ω,

$$\sum_{x \in \Omega} \sum_{y \in \mathcal{N}(x)} J_{x \to y}$$

internal flux terms cancel pairwise. Only boundary terms remain. Thus total participation changes only via boundary coupling.

Closure Ordering

Closure ordering defines the sequence by which participation updates occur across the substrate.

Definition 3 (Closure Ordering)**.** *Closure ordering is the discrete update sequence by which null participation states resolve into stabilized outcomes under coupling constraints.*

Closure ordering imposes:

- Finite update propagation speed,
- Local adjacency mediation,
- Sequential resolution under participation openness.

Finite Propagation Constraint

Let t denote discrete closure step.

Participation updates satisfy locality:

$$s_x(t+1) \text{ depends only on } \{s_y(t) : y \in \mathcal{N}(x)\}.$$

This prohibits non-local instantaneous participation transfer.

Structural Stability Conditions

Stability requires that participation cycles do not diverge unboundedly.

Energy Functional

Define discrete participation energy:

$$E = \frac{1}{2}\sum_x \sum_{y\in\mathcal{N}(x)} |\psi(y) - \psi(x)|^2 + \sum_x V(\psi(x)),$$

where V is bounded below.

Lemma 1. *If V is bounded below and transport obeys local conservation, then E remains bounded for finite substrate regions.*

Justification.

Gradient contributions are finite for finite adjacency. Bounded potential prevents unbounded local growth. Transport redistributes but does not amplify participation.

Null Stability Constraint

Null states must not dominate indefinitely.

Proposition 3. *A substrate configuration with persistent unresolving null is unstable under finite coupling.*

Justification.

Null represents participation openness. Finite coupling probability implies eventual resolution. Infinite null persistence contradicts participation capacity.

Minimal Constraint Theorem

Theorem 1. *Under local transport conservation, finite closure ordering, and bounded participation potential, the QUADS substrate admits stable non-trivial configurations.*

Sketch of Argument. - Uniform participation distribution satisfies conservation. - Localized perturbations diffuse under transport flux. - Bounded potential prevents runaway amplification. - Null states resolve under coupling. - Thus persistent structured configurations are admissible.

Remark 2. *Transport conservation, closure ordering, and null-resolution together guarantee that emergence occurs without ontological violation of the Logical Flow.*

Non-Primitive Dimensional Structure

Dimensionality is not assumed as primitive within QUADS. No coordinate manifold or background metric is introduced at the substrate level.

Dimension must therefore arise from stabilized participation structure.

Angular Lift and Stratified Depth Illustration

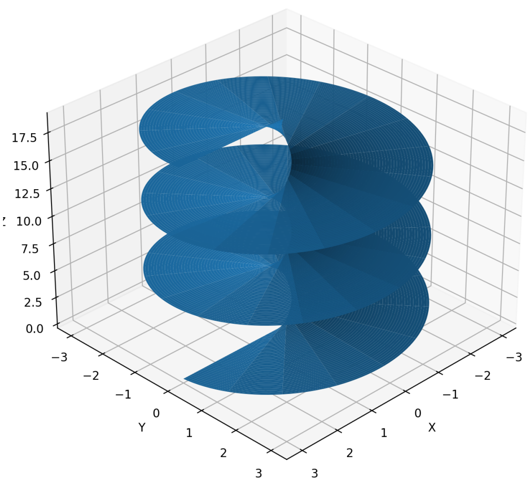
To illustrate non-primitive dimensional emergence, consider a continuous angular lift defined by:

$$(x, y, z) = (r\cos\theta,\ r\sin\theta,\ c\,\theta).$$

Lateral angular rotation in the (x, y) plane generates inward expansion through monotonic accumulation in z.

This construction demonstrates how effective three-dimensional structure can arise from angular progression without presupposing a primitive three-dimensional manifold.

Depth is therefore not an independent coordinate. It is an accumulated participation layer indexed by angular transport.



Angular lift surface illustrating stratified depth accumulation from continuous lateral angular progression. The vertical axis represents participation-layer indexing rather than primitive spatial extension.

Adjacency Graph Structure

Let $G = (V, E)$ represent the substrate graph, with finite adjacency degree:

$$k(x) = |\mathcal{N}(x)|.$$

Connectivity is purely relational. No geometric embedding is assumed.

Relational Scaling

Effective dimensionality emerges from participation scaling properties across adjacency layers.

Define the r-step neighborhood of x:

$$B_r(x) = \{y \in V \mid d_G(x, y) \leq r\},$$

where d_G is graph distance.

Define cumulative participation volume:

$$V(r) = \sum_{y \in B_r(x)} D(y).$$

Scaling Relation

If participation stabilizes into a regular pattern, then for sufficiently large r:

$$V(r) \propto r^{d_{\text{eff}}},$$

where d_{eff} is the effective relational dimension.

Definition 4 (Effective Relational Dimension)**.** *The effective relational dimension d_{eff} is defined by the scaling exponent:*

$$d_{\text{eff}} = \lim_{r \to \infty} \frac{\log V(r)}{\log r},$$

provided the limit exists.

Dimensionality is therefore a scaling property of stabilized participation distributions.

Spectral Characterization

An alternative dimensional characterization arises from the discrete Laplacian operator.

Define graph Laplacian:

$$(\Delta\psi)(x) = \sum_{y \in \mathcal{N}(x)} (\psi(y) - \psi(x)).$$

The spectral density of Δ determines diffusion behavior.

If return probability $P(t)$ under discrete diffusion satisfies:

$$P(t) \sim t^{-d_s/2},$$

then d_s defines spectral dimension.

Remark 3. *In stabilized participation regimes, d_s and d_{eff} coincide under uniform adjacency conditions.*

Dimensional Emergence from Stabilized Participation

Dimension is not assigned. It emerges from repeated participation stabilization.

Null-resolution cycling produces persistent transport patterns.

Persistent transport patterns generate relational invariants.

Relational invariants determine scaling behavior.

Scaling behavior defines dimension.

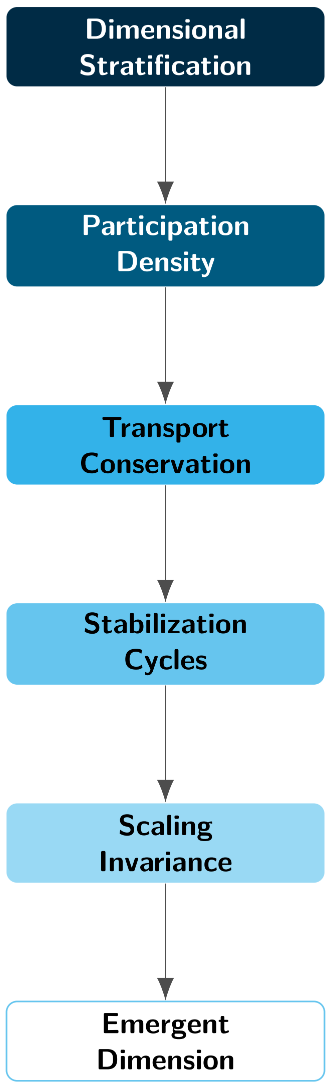
Proposition 4. *Dimensionality is an emergent statistical invariant of stabilized participation distributions.*

Justification.

Without stabilized participation, no regular scaling law exists.

Without scaling law, no consistent dimension can be defined.

Therefore dimension depends on prior participation stabilization.



Dimensional Stability Constraint

Dimensional structure remains stable only if:

1. Transport conservation holds,
2. Participation density remains bounded,
3. Null-resolution cycles do not dominate globally.

Violation of these conditions leads to:

- Dimensional collapse (vanishing scaling),
- Dimensional divergence (unbounded growth),
- Fractal or non-integer scaling regimes.

Remark 4. *Non-integer effective dimension corresponds to irregular stabilization patterns and non-uniform participation clustering.*

Conceptual Consequences

The Quantum Angular Density Substrate (QUADS) establishes a transport-first ontology in which no geometric manifold is assumed at the primitive level.

The following consequences follow directly from the preceding sections:

1. Participation is the primitive physical quantity.
2. Null is the necessary condition for transformation.
3. Resolution generates persistence.
4. Persistence under repetition generates regularity.
5. Regularity yields relational invariance.
6. Invariance permits geometric describability.
7. Dimensionality is a scaling invariant of stabilized participation.

Thus geometry is not ontologically primary. It is a secondary structural description of stabilized participation history.

Formal Summary

QUADS I establishes:

- A discrete substrate defined by lateral adjacency and inward depth.
- A non-negative participation density $D(x)$.
- Participation amplitude $\psi(x) = \sqrt{D(x)}$.
- Transport conservation across adjacency relations.
- Closure ordering governing state transitions.
- A tri-state x-register ontology $\{0, \text{null}, 1\}$.
- Stability conditions ensuring bounded participation evolution.
- Emergent dimensionality derived from relational scaling laws.

No metric tensor, curvature scalar, or geometric manifold is introduced at this stage.

The substrate-level ontology is therefore logically prior to geometry and gravitational reconstruction.

Glossary

Quantum Angular Density Substrate (QUADS) The primitive transport-first physical substrate defined by participation density.

Participation Density $D(x)$ A non-negative scalar representing active inward engagement at substrate location x.

Participation Amplitude $\psi(x)$ The square-root representation of participation density.

x-Register Minimal discrete participation state carrier with tri-state structure $\{0, \text{null}, 1\}$.

Null Structurally open participation state permitting alternative resolution.

Resolution Selection of outcome from participation openness under coupling.

Closure Ordering Sequential update rule governing null-to-persistence transitions.

Transport Conservation Local redistribution of participation without net creation or annihilation.

Effective Relational Dimension Scaling exponent characterizing stabilized participation growth across adjacency layers.

References

[1] James Johan Sebastian Allen, *Logarithmic Dimensional Shift: From Branch Cuts to Stratified Manifolds*, 2026.

[2] James Johan Sebastian Allen, *Logarithms as Geometric Charts: From Riemann Surfaces to Discrete Lattice Lifting*, 2026.

[3] James Johan Sebastian Allen, *Six-Fold Morphogenics: A Structural Theory of Form from Julia-Type Fields to Living Systems*, 2026.

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Document Timestamp and Provenance

This document is part of Pattern Field Theory (PFT) and the Quantum Angular Density Substrate (QUADS) framework. It defines the primitive ontological substrate, participation density formalism, transport conservation, closure ordering, x-register structure, and emergent dimensional scaling principles foundational to QUADS I–IV.